Clear Evidence of a Continuum Theory of 4D Euclidean Simplicial Quantum Gravity *

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Four-dimensional (4D) simplicial quantum gravity coupled to both scalar fields (N_X) and gauge fields (N_A) has been studied using Monte-Carlo simulations. The matter dependence of the string susceptibility exponent $\gamma^{(4)}$ is estimated. Furthermore, we compare our numerical results with Background-Metric-Independent (BMI) formulation conjectured to describe the quantum field theory of gravity in 4D. The numerical results suggest that the 4D simplicial quantum gravity is related to the conformal gravity in 4D. Therefore, we propose a phase structure in detail with adding both scalar and gauge fields and discuss the possibility and the property of a continuum theory of 4D Euclidean simplicial quantum gravity.

1. Introduction

4D Euclidean Simplicial Quantum Gravity (4D Eucl. SQG) has been investigated from several points of view. As a first step, the phase diagram in case of pure gravity has been studied numerically. Especially, it has been shown that the phase structure in case of pure gravity has two distinct phases, the crumpled phase and the elongated phase, and the phase transition between the two phases is shown to 1st order. Moreover, the string susceptibility exponent ($\gamma^{(4)}$) takes positive value at the critical point. As a result, it is difficult to construct to a continuum theory.

For the next stage, the case of adding gauge matter fields has been intensively investigated[1, 2]. Then, the phase structure changes drastically with adding gauge matter fields. the smooth (crinkled) phase with negative $\gamma^{(4)}$ was found even for $N_A \geq 1$[2]. And the phase transition between the crumpled phase and the smooth phase has been shown to be continuous[3]. We consider that the dynamical triangulated manifold is stable with adding some matter fields. It is expected that the 4D Eucl. SQG coupled to matter fields reaches to the continuum theory.

Numerical results indicate that the 4D Space-time becomes stable with adding the matter fields. It is supported by the analytical calculations of 4D conformal gravity[4–6]. We emphasize that the analytical calculations from 4D conformal filed theory are similar to numerical ones in case of adding matter fields. In this article, we would like to clarify the relation between 4D conformal field theory and 4D Eucl. SQG, quantitatively.

We consider that $\gamma^{(4)}$, as well as $\gamma^{(2)}$ in 2D, is given by a function of the number of matter fields,

$$\gamma^{(4)} = \gamma^{(4)}(N_X + 62N_A). \qquad (1)$$

Coefficients for the number of fields relates to the central charge of the model and can be calculated exactly from 4D conformal field theory. In order to investigate the relation between 4D Eucl. SQG and the quantum field theory of gravity, we estimate the matter dependence for $\gamma^{(4)}$ by numerical simulation with adding scalar fields (N_X) and

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gauge fields (N_A).

2. Model

The action S is given on the 4D simplicial manifold as,

$$S = S_G + S_A + S_X, \tag{2}$$

where S_G, S_A and S_X denote the action of the gravity part, the U(1) gauge fields (A) and scalar fields (X), respectively. For the gravity part, we use the discritized Einstein-Hilbert action in 4D,

$$S_{EH}[\kappa_2, \kappa_4] = \kappa_4 N_4 - \kappa_2 N_2, \tag{3}$$

where N_i denotes the number of i-simplex. The two parameters, κ_2 and κ_4, correspond to the inverse of the gravitational constant and the cosmological constant, respectively. The action,

$$S_A = \sum_{t_{ijk}} o(t_{ijk})(A_{ij} + A_{jk} + A_{ki})^2, \tag{4}$$

corresponds to that of gauge matter fields, where A_{ij} is the non-compact gauge field defined on the link l between vertices i and j with the sign convention, $A_{ij} = -A_{ji}$. The weight factor $o(t_{ijk})$ is given by the number of 4-simplices sharing the triangle t_{ijk}. We also add massless scalar fields,

$$S_X = \sum_{ij}(X_i - X_j)^2, \tag{5}$$

where X_i is the scalar field defined on the vertex i. Then the partition function is given as,

$$Z(\kappa_2, \kappa_4, N_A, N_X) = \sum_T e^{-S_G(\kappa_2,\kappa_4)}$$

$$\prod_{N_A,N_X} \left(\int \prod_{l \in T} dA_l e^{-S_A(A_l)} \int \prod_{i \in T} dX_i e^{-S_X(X_i)} \right). \tag{6}$$

We sum over all 4D simplicial triangulation (T) in order to carry out a path integral about the metric. Here, we fix the topology with S^4. Measurements are made at every 100 sweeps.

3. Matter Dependence of 4D Simplicial Quantum Gravity

In this section, we report on the matter dependence of $\gamma^{(4)}$. We measure $\gamma^{(4)}$ by using the MINBU method. In Fig.1, we plot $\gamma^{(4)}$ for various numbers of scalar fields versus κ_2. It is clear that $\gamma^{(4)}$ becomes negative with adding scalar fields, which is similar to the case of adding gauge fields.

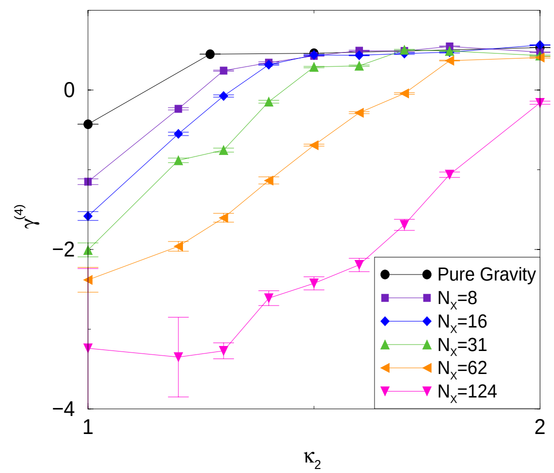

Figure 1. The string susceptibility exponent $\gamma^{(4)}$ plotted for various numbers of scalar fields (N_X) versus the coupling constant (κ_2).

Then, the matter dependence of $\gamma^{(4)}$ is shown. In Fig.2, we plot $\gamma^{(4)}$ versus the numbers of scalar fields (N_X). In order to see the matter dependence of $\gamma^{(4)}$, we compare two different cases which are the case of no scalar fields ($N_A = 0$) and the case of adding one gauge field ($N_A = 1$). For both of these cases, we measure $\gamma^{(4)}$ for various numbers of scalar fields and plot both of the results for $N_A = 0$ and $N_A = 1$, respectively in Fig.2. As compared both of the results, the case of $N_A = 1$ can be corresponded to that of $N_A = 0$ with shifting $N_X \to N_X + 62 \times 1$. Furthermore, we also plot the results of $N_A = 1$ and $N_A = 2$ with no scalar field $N_X = 0$. Both of these results are placed on the results of $N_A = 0$ with varying numbers of scalar fields (N_X).

Our numerical results suggest that $\gamma^{(4)}$ is the function of the number of scalar fields (N_X) and gauge fields (N_A),

$$\gamma^{(4)} = \gamma^{(4)}(N_X + 62 N_A). \tag{7}$$

Moreover, the coefficient 62 corresponds to the result of 4D conformal field theory.

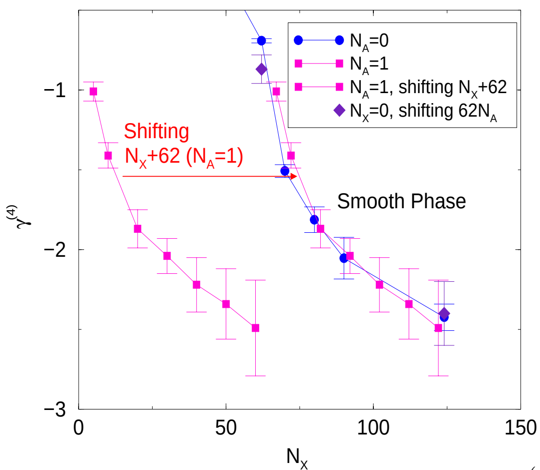

Figure 2. The string susceptibility exponent $\gamma^{(4)}$ plotted versus numbers of scalar fields (N_X).

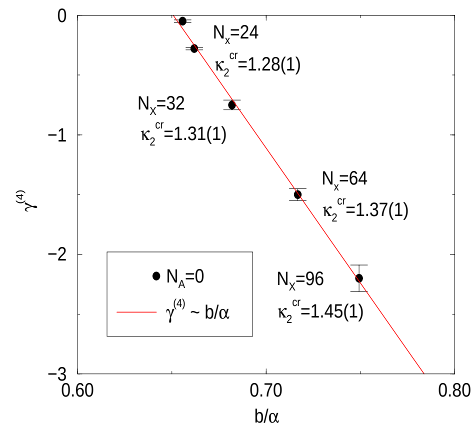

Figure 3. The string susceptibility exponent $\gamma^{(4)}$ plotted versus b/α.

4. Summary and Discussions

Let us summarize and discuss the main points made in the previous section. We measure γ^4 for various numbers of scalar fields (N_X). In Fig.2, we show that the matter dependence of $\gamma^{(4)}$ is given as, $\gamma^{(4)} = \gamma^{(4)}(N_X{+}62N_A)$. The coefficients 62 is equivalent to the result of 4D conformal field theory. Numerical results suggest that 4D Eucl. SQG is related to the conformal gravity in 4D.

Furthermore, we compare our numerical results with BMI formulation to describe the quantum field theory on a non-dynamical background metric theory of gravity with the dynamics of the traceless mode as well as the conformal mode in 4D[5,6].

In order to compare the results of BMI formulation with 4D Eucl. SQG, we focus the matter dependence of $\gamma^{(4)}$. From BMI formulation, $\gamma^{(4)}_{BMI}$ can be estimated as,

$$\gamma^{(4)}_{BMI} \propto \frac{b}{\alpha}, \tag{8}$$

with the ambiguity of the volume definition. The quantity b plays the role of the central charge in BMI formulation and α is the dimension of the cosmological constant. We show the numerical result at the near critical point in Fig.3

Numerical results show the fact that $\gamma^{(4)}$ is proportional to b/α. It corresponds to the estimation from BMI formulation. Then, $\gamma^{(4)}$ is given as the function of the quantity b that plays the role of the central charge in 4D. It is the same situation as $\gamma^{(2)}$ is given as the function of the central charge c in 2D.

Numerical results suggest that 4D Eucl. SQG coupled matter fields relate to the conformal invariance in 4D and BMI formulation. It should be clear that a continuum theory of 4D Eucl. SQG depends on dynamics of the traceless and conformal mode. Moreover, we expect that the properties of 4D quantum gravity are similar to those of 2D quantum gravity.

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